SPEED AND SHAPE OF AIR BUBBLES IN WATER

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The motion of a gas bubble in a liquid is governed by changes in bubble shape. The deformation at breakaway is determined by the reaction (density) of the liquid, the surface tension, and the size of the nozzle. There are subsequently damped oscillations unrelated to the motion, with a gradual return to spherical, which increase the resistance and reduce the rate of rise. The instantaneous rate of rise is thus determined by the breakaway diameter and the time from the start of the motion. Formulas are given for the rise rate u and the shape.

Published formulas [1–5] for u apply to relatively large bubbles and are usually deduced on the assumption of potential flow around the bubble, while the deformation is caused by the resistance. The various formulas give substantially different results. For instance, if we use the equivalent diameter d (the diameter of the sphere having the same volume), we get the following values of u (m/sec) for $d = 0.718 \cdot 10^{-2}$ m in water: from [1, p. 41] 0.166, from [2] 0.189, from [3] 0.391, and from [4] and [5, p. 98] 0.224. Some of these formulas [1, 2] give u as a function of diameter, while others [3–5] give the dependence only on the physical properties of the liquid, and all relate to steady-state motion of a bubble of constant volume, whereas the actual motion is not steady-state [6] and u is related to the shape [7].

Here we report some results for bubbles of initial diameter 0.58–$0.64 \cdot 10^{-2}$ mm in distilled water.

1. The tests were performed in a transparent column (Fig. 1) 2 m high and 0.2×0.2 m in cross section. A single bubble was formed by smoothly injecting air from a medical syringe 13 into the nozzle 2 attached to the bottom of the column 1.

We measured u as the time taken to pass between fixed points. The column was divided into parts by narrow ($0.2 \cdot 10^{-2}$ m) horizontal light beams from the sources 3 and stops 4, which lay at 0.01, 0.292, 0.572, 0.842, 1.138, 1.418, 1.700, and 1.974 m from the nozzle. Each beam was passed by a lens 5 to a photoresistor 6 (type FS-K1), which was connected in the circuit of an N-700 galvanometer. The current pulses were recorded on photographic paper moving at 0.04 m/sec, which also recorded 10- and 50-Hz time marks, which indicated the time to ±0.01 sec. The results for u had a relative error of 1–3.5%.

The bubble shape and size were determined photographically at heights of 0.01, 0.842, and 1.70 m. Camera 10 had its shutter open and received light from the flash lamp 9, which was triggered by a fast relay operated by the amplified current pulse from a photoresistor.

The shape change at the start was examined with a motion-picture camera running at 26.13 ± 1.1 frames/sec (0.038 sec between frames).

The height b and frontal diameter a were measured with the photographs enlarged 10 times, the magnification being determined from the image of a cylinder of known size in the plane of motion. We examined photographs of 220 bubbles at various heights h of the center above the nozzle. The tests were done with air and water at 20 ± 1° C.

2. Photographs in two orthogonal planes parallel to the direction of motion showed that the bubbles were nearly flattened spheroids of rotation. Frames 1–3 in Fig. 2 are for $d_0 = 0.61 \cdot 10^{-2}$ m and h = 170.0 cm, m = 1.91; h = 84.2 cm, m = 2.02; and h = 1.0 cm, m = 2.30, in which $m = a/b$ is the flattening. The bubble expanded as it rose, and m fell (Table 1), while the shape deviated from spheroidal, which was confirmed by the increase in the difference between the actual volume

$$V = \frac{V_1}{N} \frac{101\ 325}{P + (H - h) g\rho} \tag{2.1}$$

and the value calculated for an ideal spheroid. Table 1 gives the relative value of this difference δ. In (2.1) V_1 is the volume (m^3) of the air in the syringe (at 293° K and 101.325 kN/m^2), N is the number of bubbles produced from this, P is the pressure (N/m^2) at the free surface, H is the height (m) of the liquid in the column, ρ is density (kg/m^3), and g is the acceleration due to gravity (m/sec^2).

The m for a given h had very nearly a Gaussian distribution.

The δ and m of Table 1 are means from 15–36 measurements for a given nozzle diameter d_0.

The motion-picture frames of Fig. 3 show that the bubble becomes greatly deformed in the initial part of its motion.

3. Table 2 gives values for the time τ for passage at various h (means from 8–15 measurements for each d_0), h being reckoned from the 0.01-m level. Least-squares processing gave h(τ), which was differentiated to give u, which decreased slowly as follows for τ < 8 sec:

$$\begin{aligned} d_0&=0.45\cdot 10^{-2}\ \text{m} & u&=23.53\cdot 10^{-2}-0.13\cdot 10^{-2}\,\tau \\ d_0&=0.61\cdot 10^{-2} & u&=23.65\cdot 10^{-2}-0.09\cdot 10^{-2}\,\tau \\ d_0&=0.78\cdot 10^{-2} & u&=23.15\cdot 10^{-2}-0.05\cdot 10^{-2}\,\tau \end{aligned} \tag{3.1}$$

Then (3.1) was used with the mean τ and corresponding h (Table 2) to derive u(h). The current diameters d were used to plot u(d) (Fig. 4). There was no single relation between u and d, but d_0 had a slight but quite distinct effect on u. A published u(d) curve [8] has been re-

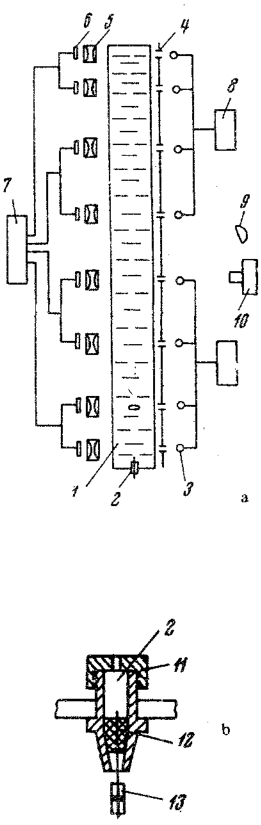

Fig. 1. 1) Column, 2) nozzle, 3) light source, 4) stop, 5) lens, 6) photoresistor, 7) oscilloscope, 8) voltage stabilizer, 9) flash lamp, 10) camera, 11) nozzle head, 12) rubber bung, 13) syringe.

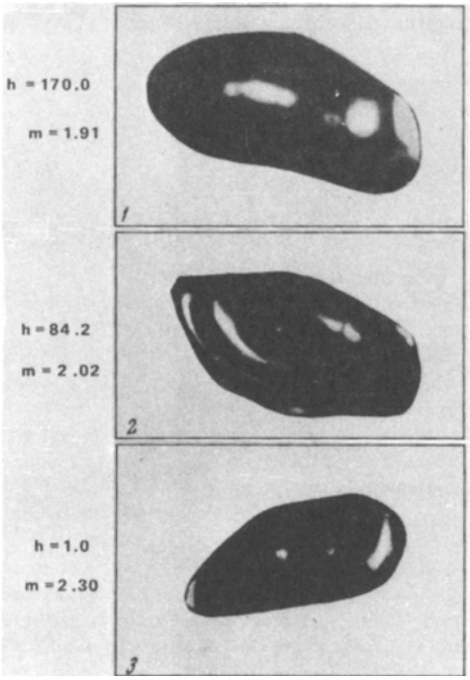

Fig. 2

produced in the Soviet literature [9] but does not describe the motion of an individual bubble, since it is derived from the mean u for bubbles of various initial d.

The observations on bubble shape in relation to d indicate that flattening increases as the bubble rises and the volume increases.

4. Bubble breakaway is fairly complicated [10], but the breakaway diameter d_* can be determined rather accurately from equality of the upthrust and the surface-tension force:

$$d_* = \left(\frac{6\, d_0 \sigma}{g \Delta\rho}\right)^{1/3}, \tag{4.1}$$

in which σ is the surface tension.

In our tests, d_* from (4.1) differed from d at 0.01 m by less than 10%:

$$\begin{aligned} d_0 \cdot 10^2 &= 0.45 \quad 0.61 \quad 0.78 \\ d_* \cdot 10^2 &= 0.585 \quad 0.648 \quad 0.704 \\ d \cdot 10^2 &= 0.583 \quad 0.619 \quad 0.643 \\ \delta\% &= 0.5 \quad 4.7 \quad 9.5 \end{aligned}$$

5. The largest deformation m_* after breakaway can be deduced from equating the work $R\delta b$ done by the external deforming force to the surface tension energy $\sigma\delta F$:

$$R\delta b + \sigma\delta F = 0. \tag{5.1}$$

If the bubble is deformed from a sphere of diameter d_* to a spheroid, $\delta b = d_* - b$, while the surface area change δF, referred to δb, is

$$\frac{\delta F}{\delta b} = \pi d_* f(m_*) = \pi d_* \frac{2^{1/2} m_* - m_*^{2/3}(m_*^2 + 1)^{1/2}}{2^{1/2}(m_* - m_*^{1/3})}. \tag{5.2}$$

The deforming force equals the upthrust on the bubble:

$$R = V_* \, g\Delta\rho, \tag{5.3}$$

and the equation for the maximum degree of deformation becomes

$$f(m_*) = -\frac{g\Delta\rho}{6\sigma} d_*^2; \tag{5.4}$$

i.e., there is a single-valued relation between m_* and d_* for a given gas-liquid system.

For d_0 = 0.0045, 0.0061, 0.0078 we calculated from (5.4) the results m_* = 2.15, 2.68, 3.16, which were close to the observed m = = 1.80, 2.30, 2.78 at 0.01 m.

Table 1

$h \cdot 10^2$	$d_0 \cdot 10^2$			δ %
	0.45	0.61	0.78	
1.0	1.80	2.30	2.78	+6.0
84.2	1.70	2.02	2.72	−18.6
170.0	1.70	1.91	2.17	−27.4

Calculations from (5.1) conflicted with experiment, as they predicted that u and m increase, so we are forced to suppose that m changes because of oscillations related to the initial deformation. The observed changes in shape described by m represent the principal (low-frequency) oscillations. The relation of m to τ was

$$m \approx m_* - 0.08\,\tau \quad (\tau < 7.5 \text{ sec}). \tag{5.5}$$

The oscillations are slowly damped, and it is not permissible to extrapolate (5.5) to times greater than 7–8 sec. The period of oscillation is $T_0 \propto 1/\nu$ [11, p. 807]. Then for liquid steel ($\nu = 3.2 \cdot 10^{-7}$ – $- 8.0 \cdot 10^{-7}$ m^2/sec) the constant in (5.5) should be somewhat less (0.03–0.06).

6. The acceleration was very small even at 0.01 m from the nozzle, and the resistance associated with this acceleration (calculated for potential flow) was only 1–2% of the total resistance R, so the latter can be calculated for uniform motion of a solid:

$$R = C\rho u^2/2\; S_a, \tag{6.1}$$

in which S_a is the area of the middle section.

With R equal to the upthrust we get the resistance coefficient as

$$C = 0.0003\, N_{\mathrm{Re}}^{1.18} m^{-1.15} \quad (N_{\mathrm{Re}} = ua/\nu). \tag{6.2}$$

The Reynolds number N_{Re} was 1760–2300 in these experiments. Equation (6.2) describes the observed C to ±3%.

It follows from (6.1) and (6.2) that the resistance to a rising gas bubble increases roughly as u^3 and increases as the bubble rises because V increases and m decreases. The observed resistance coefficients are 0.8–1.18 and agree as to magnitude and dependence on N_{Re} with results [12] for CO_2 in water, while the m dependence is qualitatively the opposite to that for solid ellipsoids [13] for N_{Re} of 1.0–$2.5 \cdot 10^5$. Unfortunately, nothing has been published on C(m) for solid ellipsoids for lower N_{Re}.

We again take R as equal to the upthrust and use the equivalent diameter $d = a/m^{1/3}$ to get from (6.1) and (6.2) that

$$u = 121.50\nu^{0.37} \frac{m^{0.03}}{d^{0.06}}, \tag{6.3}$$

which describes the observed u to 0.2–2%.

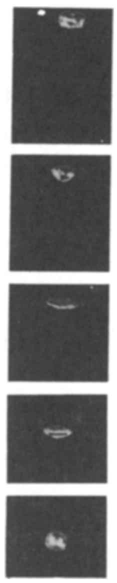

Fig. 3

Table 2

$h \cdot 10^2$	$\tau \pm \sigma_\tau$		
	$d_0=0,45 \cdot 10^{-2}$	$d_0=0,61 \cdot 10^{-2}$	$d_0=0,78 \cdot 10^{-2}$
28.20	1.16±0.014	1.18±0.029	1.21±0.006
56.20	2.36±0.029	2.39±0.027	2.41±0.008
83.20	3.58±0.030	3.57±0.029	3.68±0.012
112.80	4.89±0.035	4.89±0.029	4.85±0.023
140.80	—	6.00±0.030	—
168.85	7.35±0.039	7.25±0.031	7.36±0.052
196.40	8.51±0.046	—	—

The low frame speed of the camera did not allow us to determine the instantaneous velocity and acceleration between the nozzle and first monitor level, but simple calculations can reveal the motion in this part.

Immediately after breakaway, where viscous forces are not yet present, we can consider the motion as accelerated in an ideal liquid, the initial acceleration being

$$\frac{du}{d\tau} = g\,\frac{\Delta\rho}{\rho}\,\frac{1}{K}, \tag{6.4}$$

in which K is the adjoint-mass coefficient for a spheroid,

$$K = 0.625\ m - 0.125 \quad (1 < m < 6). \tag{6.5}$$

Then (6.4) with $m = m_*$ shows that the acceleration at this instant is of the general order of g (5.33–8.04 m/sec^2), but u has nearly reached a steady value even at 0.01 m (1–2 times d_*), and the acceleration has become small and negative. Hence u must have a maximum somewhere in this short section.

Integration of (6.4) with $u_* = 0$ gives a value of u at 0.01 m greater than the observed u; for bubbles with d_* of $0.585 \cdot 10^{-2}$, $0.648 \cdot 10^{-2}$, $0.704 \cdot 10^{-2}$ m, the ratios of the calculated u to the actual u were 1.48, 1.24, 1.12. Also, the calculated results hardly alter if we put $m = m_* =$ const in (6.4) or use the $m(\tau)$ of (5.5). If to (6.4) we add a term for the velocity-dependent resistance via (6.1) and (6.2), the calculated u is still much the same; i.e., the proportion of the total resistance related to the velocity does not have an appreciable effect on the motion.*

The motion in the initial part is thus determined by the inertial resistance of the liquid. The acceleration decreases rapidly as the bubble rises, so the K of (6.4) should increase substantially from the start, evidently because of oscillations after breakaway, which are clearly seen on photographs. An approximate estimate for $\langle m \rangle$, the mean m in the acceleration section, can be made from (6.4) and (6.5) from the equality of the actual and calculated u at 1 cm from the nozzle; the result is $\langle m \rangle = 4.0$.

7. The acceleration part is very short, and the mean u over a height H, namely $\langle u \rangle$, can be deduced from a simplified scheme, with the assumptions that:

1) the bubble is instantaneously deformed to m_* at breakaway,

2) the subsequent motion is determined by the resistance given by (6.1).

Then (6.3) is integrated with $u = dh/d\tau$ and the $m(\tau)$ given by (5.5), the volume being determined solely by the hydraulic pressure, i.e.,

$$d = d_*\left[\frac{B}{B-h}\right]^{1/3}, \qquad B = H + \frac{P}{g\rho}, \tag{7.1}$$

which gives

$$\Phi(H) = A\psi(T), \qquad \Phi(H) = \frac{B^{0.98} - (B-H)^{0.98}}{0.98} \tag{7.2}$$

$$A = \frac{121.50\nu^{0.37}}{d_*^{0.06}B^{0.02}}, \qquad \psi(T) = \frac{m_*^{1.03} - (m_* - 0.08T)^{1.03}}{0.082}. \tag{7.3}$$

For m_* of 1–4 and $T \le 15$ sec we have $\psi(T) \approx T$, so error in estimating m_* does produce a substantial error in $\langle u \rangle$ (rounding of the power to 1 in (7.2) for $\Phi(H)$ leads to an error of 15% in u). Then the time T to rise the full height H of the column of liquid is

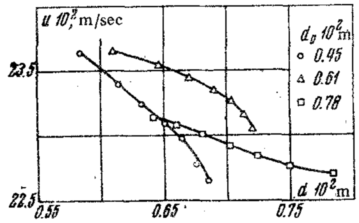

Fig. 4

$$T = \frac{\Phi(H)}{A}, \tag{7.4}$$

while

$$\langle u \rangle = \frac{H}{T}. \tag{7.5}$$

This calculation gives a very rough estimate of the speed near the nozzle; e.g., at 0.01 m the calculated u is 1.7 times the actual one. However, the result is reasonably accurate for larger distances, as the observed T at 0.842 and 1.70 m differ from the calculated ones by only 1.0–1.5%. Results for u in agreement with experiment are given by Byakov's formula [4] and Frank-Kamenetskii's [5, p. 98] if we insert the empirical coefficient $\xi = 1$ in these formulas; if $\xi = 1.3$, Frank-Kamenetskii's formula gives a result about 15% too low.

We calculated $\langle u \rangle$ in this way for inert gases in liquid steel (H = 1.0 m) as functions of ν ($3.2 \cdot 10^{-7}$ to $8.0 \cdot 10^{-7}$ m^2/sec) as 0.15–0.21 m/sec, which are less by factors of 3–4 than the 0.59 m/sec assumed by Pavlov and Popel [14].

The erroneous conclusion has been drawn [6, 7, 15] that m and u are uniquely related to d, evidently from incorrect analysis of the experiments (bubbles of various volumes were observed at a fixed distance from the nozzle and the results applied to single bubbles).

We find that the drag on a bubble is proportional to u^3, so the models of [1–3], which assume a potential distribution for the speeds near the bubble, are incorrect, as they give a linear dependence on u.

There is no rigorous theory of the motion of gas bubbles in liquids, so only semiempirical methods can be used in calculations, as here.

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*Formulas (6.1) and (6.2) give R much too small for u small; but even if we assume $R = 2\pi d\mu u$ [11, p. 753] for $N_{Re} = 1$, R is still negligibly small relative to the upthrust ($R = 6.4 \cdot 10^{-7} q\Delta\rho$).

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